Collusion in Procurement Auctions: Structural Estimation of Bidders Costs*

Maria Jakobsson†

February 1, 2007

Abstract

In this paper, the aim is to analyse the existence of collusion in asymmetric asphalt-procurement auctions. Firms that behave competitively should have private costs that are independent, conditional on available firm- and auction-specific information. The hypothesis of conditional independence can be tested and if it is rejected, a possible explanation is collusion. Using a constrained strategy equilibrium concept in solving for equilibrium bid strategies and firms' private costs makes it possible to test the hypothesis of conditional independence while at the same time controlling for firms' strategic considerations. The analysis is based on bid data from procurement auctions carried out in Sweden during the 1990's. The findings are that the hypothesis of conditional independence can be rejected for about half the firm-pairs that are tested. Given that the model is correctly specified, this suggests that collusive behaviour is plausible in the investigated market.

Keywords: Auction data, Constrained Strategy Equilibrium, Asymmetric bidders, Collusion.

JEL classification: D44, C15.

*I am indepted to Matias Eklöf for invaluable help and support during this project. I would also like to thank Mattias Ganslandt, Pehr-Johan Norbäck, Jonas Häckner and workshop participants at the Department of Economics, Stockholm University for valuable comments on earlier versions of this paper. I gratefully acknowledge financial support from The Swedish Competition Authority and the Swedish National Road and Transport Research Institute (VTI). The author is responsible for all errors.

†Department of Economics, Stockholm University, SE-106 91 Stockholm, Sweden, Fax: +46-8-16 13 98, e-mail: maria.jakobsson@ne.su.se

1 Introduction

Collusive behaviour is a serious problem in many procurement auctions. Naturally, fighting collusion is a high-priority issue for most competition authorities. Detecting and deterring cartel activity is crucial, and a great deal of effort is thus put into finding the appropriate tools. Also in the area of research is the issue of collusion dealt with extensively. Over the years, more realistic bidding models have emerged. Researchers have acknowledged, both empirically and theoretically, that bidders' costs may differ ex ante, i.e., that firms are asymmetric. Researchers such as Porter and Zona (1993, 1999) show empirically that variables like location and managerial differences affect bidding patterns. Others, such as Maskin and Riley (2000a, 2000b) and Lebrun (1999) have set up asymmetric bidding games and proven theoretically the existence of unique asymmetric Bayes-Nash Equilibria (BNE).

In this paper, the existence of collusion in the Swedish asphalt-paving industry is analysed. Deviations from a competitive BNE with asymmetric bidders is investigated using structural-estimation techniques. More specifically, by testing a necessary and sufficient condition of a competitive equilibrium (conditional independence) in the auction model discussed by e.g. Bajari (2003), the market is screened for collusive behaviour. This condition implies that after controlling for all publicly observable information on costs, the submitted bids must be independent. If, on the other hand, it is found that bids are correlated, this may suggest that the firms have engaged in a bid-rigging behaviour.

In earlier research, the tests of conditional independence have been based on estimations of the bid-level, i.e., reduced-form estimations. See e.g. Porter and Zona (1999), Bajari and Ye (2003) and Jakobsson (2006) for some examples using reduced-form models. This research, in contrast, tests the hypothesis of conditional independence using estimates of firms' private costs. The firms' costs for carrying out paving contracts are naturally not observed by the researcher. However, cost estimates can be found by structurally estimating equilibrium bid functions using the observed bids and observable information on firms' costs.

The advantage of using costs, rather than bids, when testing the hypothesis of conditional independence is that costs do not suffer from strategic considerations. A strong firm may act strategically and scale up its bid if it knows that it faces a weak firm. If not controlled for, this type of strategic behaviour may give false indications of collusion in the tests of conditional independence. However, when deriving the firms' cost draws with structural methods, information on the competitors' cost distributions is taken into consideration. Therefore, under the assumptions of the auction model, and given that all the observable information on firms' costs is included in the model, i.e. that the model is correctly specified, the estimated cost draws should not depend on strategic considerations.

Structural models are closely connected to auction theory in that they use the theoretical assumption that the observed bids are generated by equilibrium-bid strategies. To find the firms' private costs necessary for the analysis pursued in this paper, the equilibrium-bid strategies need to be solved for. The structural-econometric literature proposes several methods to find the equilibrium-bid strategies of a first-price sealed-bid auction. The parametric approach (e.g. Paarsch (1992)) specifies the firms' joint cost distribution up to a vector of unknown parameters. Maximum likelihood and method of moments are methods that can be used to find the equilibrium strategies and the parameters that define them. However, for asymmetric bidders, it is not trivial to find the equilibrium-bid strategies. In this case, the equilibrium strategy profile is defined as a system of differential equations that rarely has a closed-form solution. These differential equations must be solved with numerical methods that can be unstable; see e.g. Bajari (2001) for a discussion on different approaches to finding the equilibrium-bid functions and the associated cost distributions when bidders are asymmetric.

During the recent couple of years, alternative approaches have appeared. A common feature of these methods is that they do not require the researcher to solve a system of differential equations. Instead, the firms' strategies and private costs can be found using other techniques. Jofre-Bonet and Pesendorfer (2003) estimate a dynamic auction game under capacity constraints. They show that the unobserved private costs can be inferred based on an estimation of the bid distribution and the first-order condition of optimal bids. Since the estimation procedure does not require solving for the equilibrium bid functions, the authors argue that it is computationally simple.

A non-parametric framework is suggested by Guerre, Perrigne and Vuong (2000). The authors use an indirect two-step procedure to identify the distribution of private costs from the observed bids. Using this procedure, the researcher is neither required to specify the cost distribution nor to compute the BNE. A sample of pseudo costs is constructed based on kernel estimates of the distributions of the observed bids. As a second step, the density of bidders' private costs is estimated nonparametrically using the pseudo costs.

Armantier, Florence and Richard (2005) propose an approach where bidders' strategies are constrained. They define the Constrained Strategy Equilibrium (CSE) concept as a BNE of a modified game where the constrained strategies are piecewise linear. Thus, to find the CSE strategies, the researcher only needs to optimize over a finite set of parameters instead of an infinite set of functions, as is the case with the BNE. The authors find that approximating BNE using the constrained strategy approach performs just as well, if not better, than solving for the differential equations generated by the first-order conditions in the extensive form game by using numerical methods. Armantier et al. (2005) also find that the method outperfoms the numerical approximation in terms of computational time.

In this paper, the constrained-strategy approach is used on bid data from asphalt-procurement auctions carried out in Sweden during the 1990's, with the aim of analysing bid behaviour. In earlier research, the constrained-strategy approach has, to my knowledge, been applied twice, not counting Armantier et al. (2005). Eklöf (2005) adopts the approach to an asymmetric first-price sealed-bid auction. The aim in this paper is to assess the social cost of inefficient contract allocation. Armatier and Sbaï (2004) apply the constrained strategy approach to approximate a complex asymmetric share-auction model.

The main findings in this paper are that the hypothesis of conditional independence, tested on firms' estimated cost draws, can be rejected for about half the firm-pairs that are tested. This suggests that collusion exists in the market and that a large group of firms are involved in the collusive behaviour. In 2003, the Swedish Competition Authority (CA) initiated legal proceedings against nine firms in the asphalt-paving market for collusive behaviour. Interestingly, for eight out of these firms the hypothesis of conditional independence can be rejected in several of the tests conducted in this paper [1]

A comparison of the results found in this analysis to the results in Jakobsson (2006), where reduced-form models are estimated on the same data set, reveals that (almost) the same group of firms are represented with significant correlation in both sets of tests. Given that the estimated model is correctly specified, this suggests that strategic considerations are not of any great importance in this particular market.

This, in turn, suggests that when analysing this particular market, reduced-form models may work equally well as the more sophisticated structural models. If this is a more general result, it indicates that reduced-form models can be used in place of structural models when testing the hypothesis of conditional independence, without loss of accuracy. Such a result would be appealing to e.g. competition authorities investigating collusive behaviour, since finding the equilibrium-bid functions associated with the structural models is technically difficult, especially when bidders are asymmetric.

However, the test used in this paper, as any test of collusive behaviour, suffers from limitations. The results found hinge on the assumptions of the auction model, and that the estimated model is correctly specified. If the firms observe information on each other's costs that is not included in the estimation, the hypothesis of conditional independence may erroneously be rejected. Tests of collusive behaviour should always be carried out with care and careful judgement should be exercised in analysing the results.

Moreover, the test cannot determine if the observed pattern is the result

[1] The ninth firm is not represented in the data used in this paper. In addition, the CA later dropped the charges against this firm.

of illegal agreements, or if it is caused by tacit collusion. The possibility also exists that, if informed of the structure of the test, a clever cartel can adapt its behaviour so that it is not observed. However, despite the fact that no test for collusion is foolproof, the analysis carried out in this paper could arguably be used as a first step in investigating the existence of collusive behaviour.

This paper contributes to the existing literature in two main respects. First, the constrained strategy-approach is applied in a new setting, using new data. Since the approach is fairly new, this application has a value in itself. Second, the hypothesis of conditional independence is tested using firms' cost draws in contrast to earlier papers where bids are used. The advantage with this is that it is possible to control for firms' strategic considerations. Moreover, since the results are very similar to those in Jakobsson (2006), where reduced-form models are estimated on the same data, the analysis carried out in this paper could be seen as a robustness test to those results.

The remainder of this paper is organized as follows. Section 2 discusses previous studies in the field of collusion. Section 3 presents the hypothesis of conditional independence and sets out a framework for the constrained strategy equilibrium. In Section 4, the procurement procedure and the data used in the analysis are presented and discussed. The details on how the structural analysis using the constrained strategy approach is carried out are described in Section 5. Section 6 presents the results from the structural estimation and the tests for conditional independence and Section 7 concludes.

2 Existing Literature

There exist several empirical papers on the subject of collusion in auctions. As noted earlier, Porter and Zona (1993, 1999) use reduced-form models to test predictions of the theoretical models, such as the condition of conditional independence. They compare the bidding pattern of firms that belong to the cartel (bid rigging is known to exist on the market) and firms that are assumed not to collude. The main finding is that the bidding behaviour of collusive firms differs from that of non-collusive firms. Tests are carried out on the rank distribution of costs and bids and the results show that for non-collusive firms, high bids are associated with high costs but this is not the cases for collusive firms.

Pesendorfer (2000) investigates bid rigging in the Florida and Texas school-milk markets. Collusion is known to exist in the markets also in this case. In line with the above papers, he finds that the bid behaviour differs between firms involved in collusion and non-collusive firms, and also that there exist different types of collusive behaviour in the different markets.

In Jakobsson (2006), the Swedish asphalt market is investigated using a reduced-form analysis. The findings are that bid rigging can be widespread in the market and that further investigation of the market is warranted.

Bajari and Ye (2003) investigate the existence of collusion in highway construction auctions by estimating both reduced-form and structural models on highway-procurement data. In the structural part of the paper, a competitive and a collusive model are estimated. The authors find that the model of competitive equilibrium performs better than the collusive model. Baldwin, Marshall and Richard (1997) also test for collusion by structurally estimating one collusive and one competitive model, based on data from timber auctions. They find that the collusive model performs better than the competitive model.

3 Theoretical Framework

3.1 The Hypothesis of Conditional Independence

Bajari and Ye (2003) identify a set of conditions about firms' cost distributions that are necessary and sufficient for a Bayesian-Nash equilibrium of a bidding game with asymmetric bidders to be competitive. One of these conditions is conditional independence. Conditional independence implies that firms' costs for carrying out a particular contract should be independently distributed. Using simple statistical methods, the hypothesis of conditional independence will be tested in this paper. In this section, I set out to explain the characteristics of conditional independence and how it can be tested.

Limiting the exposition to one single auction, assume that there exists one cost distribution for each firm participating in the auction (this follows from the assumption of asymmetric bidders). Each firm's cost distribution is defined by a vector of structural parameters θ and a vector $\mathbf{x}_i$ with firm-specific covariates, such as the distance from production plant to project site. The associated cumulative distribution function for firm i can then be written $F_i(c_i; \theta_i, \mathbf{x}_i)$. Assume further that firm i's cost draw c_i, i.e. firm i's private cost for carrying out the project, can be estimated as a function of the vector of firm-specific variables:

$$c_i = \mathbf{x}_i^{'}\beta + \varepsilon_i, \qquad i = 1, ..., n. \tag{1}$$

The error term ε_i is assumed to be normally distributed with zero expectation and constant variance σ^2. Hence, the vector of structural parameters, $\theta = (\beta, \sigma^2)$, identical for all bidders, together with the vector of firm-specific covariates, produce a unique cost distribution for each firm i. The expected cost draw from firm i's cost distribution is represented by $\mathbf{x}_i^{'}\beta$ and the variance by σ^2 (the variance is assumed to be constant across firms). Bajari and Ye (2003) show that, conditional on the firm-specific information available to the researcher, the private cost draws $\mathbf{c} = (c_1, ..., c_n)$ should be independently distributed if bidders are acting competitively. If the condition of conditional independence holds, so must the following relationship

$$F(c_{1,...,}c_n; \mathbf{x}) = \prod_{i=1}^{n} F_i(c_i; \mathbf{x}_i) \ , \tag{2}$$

where $F(\cdot)$ is the joint cost distribution for all firms participating in the auction. If, on the other hand, bidders are colluding, the above relationship is not necessarily true.

Bajari and Ye (2003) choose to test if the observed bids, rather than the unobserved costs draws, are conditionally independent. Since the submitted bids are a function of the latent costs, the relationship in (2) must necessarily hold also for the bid distributions. To test the condition in (2), the observed bid distributions can be estimated as a function of the vector with firm-specific covariates, using reduced-form models. The part of the observed bid that cannot be explained with the vector of firm-specific variables, the residual, can then be tested for independence. If bids are submitted independently, the residuals should not be correlated between bidders. If, on the other hand, correlation is found, and it is negative, this may suggest a bid-rigging behaviour, i.e., a collusive behaviour where the bidders take turns submitting the winning bid and a higher phantom bid to give the appearance of competition. In e.g. Porter and Zona (1999) and Jakobsson (2006), the hypothesis of conditional independence is tested using residuals from reduced-form bid-level estimations.

In this paper, the hypothesis of conditional independence is tested with the aim of investigating if cost distributions, derived with structural-econometric techniques from a given distribution of observed bids, are consistent with a competitive equilibrium. As will be discussed in the next section, solving for the firms' equilibrium bid strategies makes it possible to find the firms' otherwise hidden cost draws. The unexplained part of the cost draw, i.e., the difference between the cost draw derived from the observed bid, and the expected cost draw found in the structural estimation, is tested for correlation across firms.

There is potential an advantage of using costs instead of bids when testing the hypothesis of conditional independence. Firms' bid decisions may depend on other factors than the information included in $\mathbf{x}_i$. A firm may e.g. adjust the bid it submits depending on the characteristics of the firms it bids against. Firms do not know the exact costs of their competitors, but following the theoretical auction model, they are assumed to know their competitors' cost distributions. A strong bidder, in the sense of stochastic dominance, knowing that he bids against a weak bidder, may choose to scale up his bid and bid higher than expected. Similarly, for the weaker bidder facing a strong bidder, it may be strategically correct to bid lower than expected to increase the probability of winning the auction. The reduced form-models estimated in e.g. Bajari and Ye

(2003) and Jakobsson (2006) show evidence of this type of behaviour. As the transportation distance of firm $j \neq i$ increases, competition softens and firm i will raise its bid. This type of strategic behaviour is not controlled for when testing the bids for conditional independence. As a result, it may lead to false indications of bid rigging.

However, when using structural models to estimate the firms' cost draws, information on the competitors' cost distributions is taken into consideration. Therefore, given that the information the firms have on their competitors' costs is the same as the information we include in the model, i.e. the cost estimation is correctly specified, the estimated cost draws should not suffer from strategic considerations.

In line with the above discussion on bid distributions, costs should be randomly distributed for the hypothesis of conditional independence not to be rejected. If, on the other hand, the residuals show a persistent correlation pattern, the hypothesis of conditional independence typically fails.If the tests show the existence of negative correlation, this may be the result of one cost draw being systematically lower, and one cost draw being systematically higher than predicted by the estimated model. Consequently, with a correctly specified model, negative correlation could be the result of a bid-rigging scheme with one low bid, aimed at winning, and one bid exceeding the low bid.

Positive correlation could more easily be explained by unobserved characteristics that equally affect the firms' costs, e.g. changes in input prices that are not controlled for in the estimation. However, another possible explanation for positive correlation could be collusive behaviour with more than two participating firms. Consider three firms taking turns in submitting the low bid. The low bid will turn out lower, and the high bids higher than predicted by the model, and negative correlation will therefore be observed. The two high bids will both turn out to be higher than what the model predicts and will therefore be positively correlated. Therefore, findings of positive and negative correlation in combination could potentially be a sign of collusive behaviour with more than two members.

The tests for conditional independence are presented and discussed in Section 6.

3.2 The Constrained Strategy Equilibrium

Being able to test the hypothesis of conditional independence discussed in the previous subsection involves finding the firms' cost distributions, which is a question of solving for the firms' equilibrium bid strategies. However, finding Baysian-Nash equilibria in games of incomplete information may be difficult or even impossible. As an attempt to approximate the otherwise untractable Bayesian Nash equilibrium strategies, the concept of constrained strategy equilibrium (CSE) will be used in this paper. Firms' equilibrium bidding strategies are constrained in the sense that they can only take a predetermined form. Following Armantier et al. (2005) and Eklöf (2005) the strategies are constrained to be piecewise linear. This means that instead of having to solve an infinite number of differential equations, which is the case with the BNE, optimizing over the finite set of parameters that defines the equilibrium bid function is sufficient.

In this subsection, the equilibrium bid strategies using the concept of CSE are presented.[2] For a general application of the CSE and approximation theorems and existence proofs, see Armantier et al. (2005).

For the purpose of this paper, consider a single asymmetric first-price sealed-bid auction, where n firms simultaneously submit bids on an asphalt project. As implied by the auction format, the lowest bidder wins the auction and is paid its bid for carrying out the project. Before bidding starts, firm i learns its private cost for completing the contract, $c_i \in C_i \subset \mathbb{R}$. The private cost c_i can be seen as an independent draw from a distribution with cumulative distribution function $F_i(c_i|\theta_i)$ and density function $f_i(c_i|\theta_i)$, where θ_i represents a vector of firm-specific parameters defining the cost distribution. The cost draw c_i is assumed to have the same support for all i, $[\underline{c}, \overline{c}]$. This is a game of incomplete

[2] The exposition in this section mainly follows the notation in Eklöf (2005).

information, that is, firm i does not know its competitors' cost for completing the contract (c_{-i}), only the distributions from which competitors' costs are drawn. Since the model allows for asymmetric bidders, the cost distributions differ across firms (or at least they can differ, but they might also be identical).

The auction is assumed to take place within the independent private values paradigm since if firms got information on their competitors' costs, their private cost estimates would likely not be altered. Several authors use the assumption of private values when modelling procurement auctions, e.g. Porter and Zona (1993, 1999), Bajari (2001) and Bajari and Ye (2003). If firm-specific factors, such as transportation and material costs, account for the differences in cost estimates, the assumption of private costs is reasonable.

Assume that S is a set of feasible unconstrained bidding strategies. Further, assume that the set of constrained bidding strategies belongs to a subset of the unconstrained strategies, $S^{(K)} \in S$. The set of constrained strategy functions is compact and convex and indexed by a K-dimensional vector. Firm i's bidding strategy, $s_i^{(K)} \in S^{(K)}$, is a function transforming the private cost to a corresponding bid

$$\begin{array}{l} s_i^{(K)} : C_i \rightarrow B_i \\ c_i \rightarrow b_i = s_i^{(K)}(c_i), \quad i = 1, ..., n. \end{array}, \tag{3}$$

where $b_i \in B_i \subset \mathbb{R}$ is the bid submitted by firm i. Following Eklöf (2005) and Armantier et al. (2005), the constrained strategies are defined as a set of piecewise linear functions having K linear segments and predetermined kinks. Alternatively, the constrained strategies could be defined as a set of polynomials of degree K. Armantier et al. (2005) show that both polynomials of low degrees and piecewise linear functions produce very accurate approximations of the Bayesian-Nash Equilibrium (BNE), suggesting that there is no need for more sophisticated constrained strategies.[3] The authors also show that polyno-

[3] For a symmetric first price independent private values model, which has a closed form solution, the authors compare actual BNE with the CSE approximation. They find that the CSE converges to a BNE as K increases, and show graphically that for $K = 5$ the constrained strategies are hard to distinguish from the BNE strategies. They further show that for K as low as three, there is very little difference between the CSE and its unconstrained best response

mials and piecewise linear functions generally provide equally accurate results, but for a given K, they conclude that the piecewise linear functions perform slightly better.

To reduce the number of parameters that define firm i's strategy function and facilitate the estimation of equilibrium strategies, it is imposed that firms bid their cost if the cost is equal to the upper support, $s_i^{(K)}(\bar{c}) = \bar{c}$.[4] Following the above discussion, firm i's strategy function can be parametrized by a vector of slope coefficients, $\mathbf{d}_i = (d_{i1}, ..., d_{iK}) \in D \subset \mathbb{R}^K$:

$$s_i^{(K)}(c_i; \mathbf{d}_i) = \bar{c} + \sum_{k=1}^{K} d_{ik}(c_i - \tau_k)\mathbf{1}(\tau_k - c_i), \quad \forall c \in [\underline{c}, \bar{c}] \ , \tag{4}$$

where $\mathbf{1}(x)$ is an indicator function that takes the value of 1 if $x \geq 0$ and 0 otherwise. The set of slope coefficients, D, is restricted to parameter values so that the constrained strategy in equation (4) is monotonically increasing in cost, and for the associated profit to be non-negative, i.e. $s_i(c_i; \mathbf{d}_i) \geq c_i, \ \forall c_i \in [\underline{c}, \bar{c}]$. This is essential in finding the equilibrium strategies. The kth kink in the strategy function is represented by τ_k.

Firms are endowed with individual von Neumann-Morgenstern utility functions denoted by $\widehat{U}\left(\mathbf{s}^{(K)}; \mathbf{c}\right) = \left\{\widehat{U}_1\left(\mathbf{s}^{(K)}; \mathbf{c}\right), ..., \widehat{U}_n\left(\mathbf{s}^{(K)}; \mathbf{c}\right)\right\}$. To win the contract and get a positive profit, firm i must be the low bidder at the same time as submitting a bid that is higher than its cost (this follows from the restriction of D, as discussed above). Firm i's profit therefore depends both on the vector of private costs $\mathbf{c} = (c_1, ..., c_n)$ and the strategy profile $\mathbf{s}^{(K)} = \mathbf{s}^{(K)}(\mathbf{c}; \mathbf{d}) = (s_1^{(K)}(c_1; \mathbf{d}_1), ..., s_n^{(K)}(c_n; \mathbf{d}_n))$. The profit, conditional on the vector of firms' private costs, and the strategy profile, can be written as

$$\hat{U}_i\left(\mathbf{s}^{(K)}; \mathbf{c}\right) = (s_i^{(K)}(c_i; \mathbf{d}_i) - c_i)\mathbf{1}(y_i - s_i^{(K)}(c_i; \mathbf{d}_i)), \tag{5}$$

in terms of expected utility. For an asymmetric auction that has no closed form solution, the authors evaluate the quality of the CSE by comparing it to the approximation procedure proposed in Li and Riley (1999). They find that the CSE performs slightly better than the Li and Riley procedure in terms of difference in expected utility between the approximated strategy and its unconstrained best response.

[4] This boundary condition is used in the theoretical literature in characterizing the solution to the unconstrained Bayesian-Nash equilibrium, see e.g. Bajari (2001).

where $\mathbf{1}(x)$ is, as previously, an indicator function taking on the value of 1 if $x \geq 0$ and 0 otherwise, and y_i is the lowest bid submitted by a competitor, $y_i = \min_{j \neq i} \left\{ s_j^{(K)}(c_j; \mathbf{d}_j) \right\} \in B$.

The characteristics of the game; the number of firms that participate in the auction n, the joint cost distribution F, the set of strategy profiles $S^{(K)}$, and the vector with utility functions $\hat{U}$, are assumed to be common knowledge among the players.

Firm i's expected unconditional utility can be written in the following way:[5]

$$\widetilde{U}_i \left(s_i^{(K)}, \mathbf{s}_{-i}^{(K)} \right) = \int\limits_{\underline{c}}^{\overline{c}} \cdots \int\limits_{\underline{c}}^{\overline{c}} \hat{U}_i \left(\mathbf{c}, \mathbf{s}^{(K)} \right) dF(\mathbf{c}) \ , \tag{6}$$

where $F(\mathbf{c})$ represents the joint distribution of bidders' private costs. An equilibrium in constrained strategies is defined to be a strategy profile $\mathbf{s}^{*(K)} = (s_1^{*(K)}, ..., s_n^{*(K)}) \in S^{(K)}$ so that no deviation in strategy by any single player is profitable, that is, $\hat{U}_i(s_i^{*(K)}, \mathbf{s}_{-i}^{(K)}) \geq \hat{U}_i(s_i^{(K)}, \mathbf{s}_{-i}^{(K)})$, $\forall i = 1, ..., N$. The reason for using unconditional utility is that the CSE is defined in strategy space rather than action space. For games with unconstrained strategies**,** the equilibrium can be defined in either space, while this is not necessarily the case for constrained strategy games.

Since firms' costs are independently distributed – which follows from the assumption of independent private values – the joint density function can be factored into marginal distributions of c_i, $F(\mathbf{c}) = F_1(c_1)F_2(c_2)...F_n(c_n)$. Further, since firm i's utility depends on the own cost distribution and the distribution of the lowest of the rivals' bid, and not on the distributions of the other rivals' bids or costs, the unconditional expected utility can be rewritten as

$$\widetilde{U}_i \left(s_i^{(K)}, \mathbf{s}_{-i}^{(K)} \right) = \int\limits_{\underline{c}}^{\overline{c}} \int\limits_{\underline{y_i}}^{\overline{y_i}} \hat{U}_i \left(\mathbf{s}^{(K)}; \mathbf{c} \right) dG_i(y_i) dF_i(c_i) \ , \tag{7}$$

where $G_i(y_i)$ represents the distribution function of y_i, with support $\left[\underline{y}_i, \overline{y}_i \right]$.

[5] The unconditional utility functions are derived from the strategic form of the game.

As noted earlier, the CSE strategies are defined by the K kinks and the slope parameters in $\mathbf{d} = (\mathbf{d}_1, ..., \mathbf{d}_n)$. This makes it possible to define the CSE strategy profile $\mathbf{s}^{*(K)}$ in terms of the vector with slope parameters. Instead of having to find the solution to a system of differential equations associated with the BNE, the determination of the CSE is reduced to finding the equilibrium vectors with slope coefficients, $\mathbf{d}^*$, that solve the following system of non-linear equations

$$\begin{aligned} \mathbf{d}_i^* &= \arg\max_{d_i \in D} \widetilde{U}_i(s_i^{(K)}(c_i; \mathbf{d}_i), \mathbf{s}_{-i}^{(K)}(\mathbf{c}_{-i}; \mathbf{d}_{-i})) \\ &= \arg\max_{d_i \in D} \int_{\underline{c}}^{\overline{c}} \int_{\underline{y_i}}^{\overline{y_i}} (s_i^{(K)}(c_i; \mathbf{d}_i) - c_i) \mathbf{1}(y_i - s_i^{(K)}(c_i; \mathbf{d}_i)) dG_i(y_i) dF_i(c_i) \\ &\forall i = 1, ..., n. \end{aligned} \tag{8}$$

However, the expected utility functions in (8) are difficult to express analytically, which means that the system of non-linear equations must typically be solved with numerical methods. The solution to the integrals in (8) can be approximated using Monte Carlo integration techniques. In practice, this approximation involves taking the average over R independent random draws from the two distributions $G_i(y_i)$ and $F_i(c_i)$, respectively. From this, it follows that the maximization problem can be expressed as

$$\begin{aligned} \mathbf{d}_i^* &\approx \arg\max_{d_i \in D} \frac{1}{R} \sum_{r=1}^{R} (s_i^{(K)}(c_i^r, \mathbf{d}_i) - c_i^r) \mathbf{1} \left(y_i^r - s_i^{(K)}(c_i^r, \mathbf{d}_i) \right) \quad , \\ &\forall i = 1, ..., n \end{aligned} \tag{9}$$

where c_i^r represents the rth draw from $F_i(c_i)$ and y_i^r the rth draw from $G_i(y_i)$. As will be made clear in Section 5, a candidate vector with parameters that define $F_i(c_i)$ makes it possible to take draws from that distribution. The distribution of rivals' lowest bids, $G_i(y_i)$, can, however, not be found analytically but can be simulated by taking random draws from $F(\mathbf{c})$ conditional on the vector with strategy parameters $\mathbf{d}$. For an auction with four bidders, this procedure is carried out in the following way: For each bidder j, R random draws are taken from $F(\mathbf{c})$. Each cost draw r is transformed into a corresponding bid by using the candidate vector with strategy parameters $\mathbf{d}_{-i}$. The lowest of the resulting three bids from each draw round, in turn, produces a random draw from $G_i(y_i; \mathbf{d}_{-i})$.

However, since the utility function is not continuous in $\mathbf{d}_i$, standard Monte Carlo techniques cannot be used in the simulations. The fact that a small change in the slope vector may change the winners' identity in one of the R simulations, so that the utility changes in discrete steps, complicates the numerical optimization. This problem can be solved by approximating the indicator function with the logistic distribution kernel, as proposed by Armantier et al. (2005), such that $\mathbf{1}(x) \approx K_h(x) = \frac{e^{x/h}}{1+e^{x/h}}$. The maximisation problem can be rewritten as the following system of nK first-order conditions

$$\begin{aligned}\frac{\partial \widetilde{U}_i(s_i(c_i^r, \mathbf{d}_i^*), s_{-i}(c_{-i}^r, \mathbf{d}_{-i}^*))}{\partial d_{ik}} = \quad & \frac{1}{R}\sum_{r=1}^{R} \frac{\partial s_i(c_i^r, \mathbf{d}_i^*)}{\partial d_{ik}} \{K_h\left(y_i^r - s_i(c_i^r, \mathbf{d}_i^*)\right) \\ & +(s_i(c_i^r, \mathbf{d}_i^*) - c_i^r) k_h\left(y_i^r - s_i(c_i^r, \mathbf{d}_i^*)\right)\} = 0\end{aligned} \tag{10}$$

$$\forall k = 1, ..., K \quad \forall i = 1, ..., n \ .$$

Finding the CSE means finding the solution to this system. Note that there are nK equations for each auction. A detailed presentation of how the CSEs are solved is given in Section 5.

4 The Industry and Data

In this section, the characteristics of the Swedish asphalt-paving industry are presented and discussed. The awarding process is described and details on the variables used in the empirical analysis are presented.

In Sweden, the majority of the roads are maintained by the National Road Administration (RA). The RA has seven local offices, each being responsible for the maintenance in their own region. The regions independently solicit bids at the beginning of each year for the projects to be carried out during that year. The projects are awarded through first price sealed bid auctions, i.e., the lowest bidder is awarded the contract and is paid its bid. Firms are invited to participate in the bidding through advertising, but the RA also sends out invitations to the firms it considers to be potential bidders. Each contract is specified in detail in terms of how the work should be carried out and the type of material to be used. One contract may consist of several road pieces with different characteristics. After the necessary information has been made public, the firms have about two months to prepare their bids. Each project is awarded separately but the RA requires the bidders to submit bids for all contracts of interest in the region simultaneously. The projects are normally carried out during the spring and summer months.

In asphalt production, the major input is gravel. Most of the larger firms active in the market produce their own gravel, while this is normally not the case for the smaller firms. The larger firms also typically have their own asphalt production plant. Asphalt is produced by mixing gravel together with bitumen, a petrol-based substance that binds the asphalt together. The production process is identical across firms.

The data used in this paper were collected from RA minutes and contain detailed information on 536 asphalt-paving contracts. The time period investigated is 1992-2002. During this period, a total of 53 firms participated in the bidding on at least one occasion, and 27 out of those firms won a minimum of one contract. Some firms are represented during the whole period and some only in individual years. Some firms are large and nationwide and some are small and locally active. Due to the consolidation of the market that took place in the first part of the 90's, the number of bidders are fewer and larger at the end of the investigated time period than at the beginning. The total value of the contracts included in the data amounts to 2.7 billion SEK in real terms, the base year being 1990.

The collected data contain detailed contract information, i.e., information on bids, the location of the project site, the timing of the auction etc. This information was supplemented with information on the location of firms' production plants, which was used to create measures of the transportation distance between production plant and project site. Since the transportation of asphalt, and especially warm asphalt, can be complicated, the transportation distance is arguably an important factor in explaining the cost. This connection has already been established in the literature, e.g. Eklöf (2005) finds that transportation distance affects the cost of carrying out a road marking project.

The distance variable $DIST_{ia}$ is introduced in the analysis. This variable measures the shortest distance between plant and project site for firm i in auction a. Not all firms have their own production plants in each of the five regions that are represented in the data, and some firms are not represented with any production plant. For firms that own plants in the region where the auction is carried out, the closest plant location is used to calculate the transportation distance. However, if there is a plant outside the region located closer to the site, this location is used. Firms without production plants in the region are assumed to buy the asphalt from the plant located closest to the project site. The transportation distances are calculated using Euclidean distances in kilometers.[6] For each individual road included in the contract, the closest transportation distance is found, and by summing over the individual distances, the variable $DIST$ is calculated.

All permanent plants are believed to be included in the data; however, this is not the case for mobile plants. Some transportation distances are therefore found to be longer than they should be. However, since it is more expensive to use mobile than permanent plants, the longer distances could, in fact, act as a proxy for the mobile plants. Long transportation distances may also be explained by the fact that some contracts in the data include paving with cold asphalt that can be transported longer distances.

As an attempt to further control for asymmetries across firms, $RPLANT$ is included in the analysis. The variable $RPLANT_{ia}$ is an indicator variable

[6] The plant and site coordinates were found manually using maps, based on information from the RA. The distances were calculated using the following formula: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, where (x_2, x_1) and (y_2, y_1) are the plant and site coordinates, respectively.

taking the value of one when firm i holds a plant in the region where auction a is located, and zero otherwise. This variable emphasises the importance of plant location; to own a plant is not sufficient to benefit from lower transportation costs, the plant must also be located close to the site.

There are several other variables that could possibly be included in the analysis to better control for bidder asymmetries. Jofre-Bonet and Pesendorfer (2003), among others, include backlog in their estimations to capture that firms have different costs at different points in time, due to previously won uncompleted contracts. Porter and Zona (1993) control for differences in cost that are relatively permanent by including a measure of bidders' maximum capacity. Porter and Zona (1993), Bajari and Ye (2003) and Eklöf (2005) include measures of capacity utilisation at different points in time to control for differences across bidders that are not permanent. Measures of the competitors' utilisation rate are also included in some of these papers. In Eklöf (2003), a variable representing incumbency, i.e., that a firm has previously bid on a particular project, is introduced in the estimations.

For several reasons, these measures of cost differences will not be used in the analysis. One reason is that the available data are not sufficient for constructing reliable capacity measures. The data used in the analysis contain information on auctions solely carried out by the state. However, the government is not the single buyer of paving services; municipalities and the private sector also hire firms to carry out projects on which the available data contain no information. Jakobsson (2006) provides some evidence on using capacity measures constructed on insufficient data.

In addition, it is possible that capacity utilisation variables based on previous bidding behaviour are endogenous and therefore problematic to use. The incumbency variable may also be endogenous, if winning a contract in one year leads to cost savings the following year.

As mentioned earlier, the timing of the auctions is such that the bidding firms do not know if they have won a contract before they bid again. Since firms have no information on backlog or capacity utilisation at the time of bidding, these kinds of variables could be considered irrelevant in the analysis. In a more dynamic setting, using measures of backlog and capacity utilisation would be more suitable.

To control for some of the variation in bidders' cost that are not controlled for by the distance and plant variables, firm-specific dummies are included in the estimations. Regional dummies are included to control for variation in costs across regions.

Detailed information on individual auctions was collected from the procurement minutes. The contracts are generally specified in detail by the RA when it comes to how the work should be carried out and what materials to use. The number of roads included in each contract, and the kind of work each road requires, varies, however. One contract may include up to 50 different road parts that demand different kinds of treatment and maintenance. Due to difficulties in homogenising this information, contract-specific information will not be used in the analysis. To control for differences across auctions, some authors choose to include contract dummies, e.g. Bajari and Ye (2003).[7]

In this paper, an alternative approach is applied to control for unobserved heterogeneity across auctions. Relative bids are used in the analysis in an attempt to homogenise the material. The bids are normalised by dividing with the auction mean bid. Ideally, engineers' estimates of the cost of carrying out a contract should be used in the normalisation. Since no such information is available, the auction mean bid is nevertheless used.

A number of auctions were removed from the original data for different reasons. Some auctions could not be used since not all bids submitted at those auctions are observed. For a bid that is not observed, it is not possible to find the associated cost. More specifically, we cannot observe cost information for these firms. Since the bid functions for all firms in an auction are found simultaneously, and depend on information on the competitors' costs, none of the bid functions for auctions with missing cost information will be correct.

[7] Attempts have been made to include auction-specific effects in the analysis but due to difficulties in reaching convergence, auction dummies were finally not included in the analysis.

Therefore, to estimate the equilibrium strategies at a particular auction, and find the firms' cost distributions, auctions that are not complete cannot be used in the estimation.

In addition, a number of auctions with fringe bidders, in this case bidders that have participated in ten auctions or less, were dropped from the sample. The information on these bidders is not sufficient in identifying firm-specific effects.

To facilitate convergence, auctions with less than three bidders were dropped. Moreover, the auctions for which the variance from the bid mean is ≥ 20 percent were dropped due to difficulties in getting convergence. Potentially important information is lost by dropping these auctions, since large deviations in the bids may be a sign of collusion. However, structural estimation puts high demands on the data and normally requires it to appear ex ante competitive. On the other hand, finding indications of collusive behaviour after having dropped outlier auctions reasonably makes the case stronger.

After having dropped a number of auctions, following the above discussion, the resulting number of auctions included in the data amounts to 461, and the number of observations to 2440. The 75 auctions that were dropped amount to about 14 percent of the original sample. Descriptive statistics for these observations are given in Table 1 below.

Table 1. Descriptive statistics

	Mean	Std.dev	Min	Max
I. Contract characteristics and winning firms (No of obs: 461)				
Number of bids	5.29	1.26	3	10
Number of invited firms	10.20	4.37	2	27
Bid (RBID)	5.10	4.08	0.12	25.74
Mean auction bid	5.56	4.45	0.14	30.28
Relative bid (RELBID)	0.92	0.06	0.63	1.09
Distance (DIST)	230.65	285.30	4.12	2214.37
Mean auction distance	253.42	242.68	8.29	1549.48
Relative distance (RELDIST)	0.92	0.57	0.11	4.05
Dummy for plant ownership (RPLANT)	0.78	0.42	0	1
II. The full data set(No of obs: 2440)				
Bid (RBID)	5.76	4.62	0.12	32.69
Relative bid (RELBID)	1	0.08	0.63	1.41
Distance (DIST)	247.78	305.61	2	2874.6
Relative distance (RELDIST)	1	0.62	0.08	4.78
Dummy for plant ownership (RPLANT)	0.68	0.47	0	1

a. Bids are denoted in real value million SEK, 1 SEK ≈ 0.14 USD.
b. Distance is denoted in kilometres.
c. The number of invited firms are observed in 450 auctions.

5 Empirical Setup

This section discusses the details of the structural-econometric analysis and the associated reduced-form bid-level models.

5.1 Structural-Form Modelling

As noted earlier, structural estimation makes it possible to uncover the otherwise hidden parameters of the bidders' cost distributions. In the analysis carried out in this paper, firms are assumed to be asymmetric in the sense that their costs for carrying out a contract are drawn from different cost distributions. This implies that there exists one individual cost distribution for each firm and each auction. As will be explained below, the structural parameters that define the firms' individual cost distributions are found simultaneously in the iterative process. After having found firms' private costs, different kinds of analyses can be carried out. In this paper, the hypothesis of conditional independence is tested using the residuals from the structural estimation, with the aim of investigating collusive behaviour.

In this setup we assume that firms' costs are truncated normal[8] with constant

[8] Attempts have been made to use the assumption of truncated lognormal costs. However,

variance across firms and auctions, i.e.,

$$F_{ia}(c_{ia}) = F(c_{ia}; \mathbf{x}_{ia}, \beta, \sigma^2) = \frac{\Phi(\frac{c_{ia}-\mathbf{x}_{ia}^{'}\beta}{\sigma})}{\Phi(\frac{0-\mathbf{x}_{ia}^{'}\beta}{\sigma}) - \Phi(\frac{2-\mathbf{x}_{ia}^{'}\beta}{\sigma})}, \forall c_{ia} \in [0, 2], \tag{11}$$

where $\Phi(\cdot)$ represents the standard normal distribution function and $\mathbf{x}_{ia}$ is a vector of firm-specific characteristics and dummy variables. Truncation occurs far out in the tails, so the probability of truncation in the denominator is close to unity. Finding firm i's private cost distribution is a question of estimating structural parameters β and σ^2 above. If firm i's cost for carrying out the contract tendered in auction a, c_{ia}, had been known to the researcher, the parameters of the cost distribution could have been found by estimating the following equation using e.g. OLS or Maximum likelihood:

$$c_{ia} = \mathbf{x}_{ia}^{'}\beta + \varepsilon_{ia}, \tag{12}$$

where ε_{ia} is the error term including private information about firm i's cost for carrying out contract a. The error term is assumed to be normally distributed with mean zero and variance σ^2.

Since firms' costs are not known I will instead, following Armantier and Richard (2000) and Eklöf (2005), define the vector with structural parameters as the solution to a fixed-point problem.

If some appropriate starting values for the firms' cost draws are inserted in equation (12), a candidate vector of structural parameters, $\theta^{(t)} = (\beta^{(t)}, \sigma^{(t)2})$, where t identifies the estimation round, can be estimated using OLS.[9] The cost draw starting values used in the first estimation round are approximated by the observed bids. This initial assumption that firms' bid functions are to bid their costs is e.g. used in Bajari (2001). The candidate cost distributions are defined by the estimated vector of structural parameters θ and vector $\mathbf{x}_{ia}$, where $\mathbf{x}_{ia}^{'}\beta$ represents the expected cost for firm i at auction a, and σ^2 is the variance.

due to difficulties in reaching convergence in the estimation procedure, no results based on truncated lognormal costs are presented.

[9] The cost draw starting value used in the first estimation round is approximated by the observed bid. This initial assumption that firms' bid functions are to bid their costs is e.g. used in Bajari (2001).

In this setup, the observed bids and the distance variable are homogenised using the mean auction values.[10] Thus, the normalised bids take on values close to one. Consequently, in this setup, the estimated costs will also end up somewhere around unity. Therefore, for simplicity, all cost distributions have the same support, $[0, 2]$, i.e. truncation is identical across firms and auctions.

As discussed in section 3, Monte Carlo integration techniques are used in solving the system of non-linear equations in (8), i.e., in finding the vector of candidate CSE strategy slopes, $\mathbf{d}_a^{(t)}$. The integrals in (8) are simulated by taking R random draws for each firm i from $f_{ia}(c_{ia})$ and $g_{ia}(y_{ia})$, respectively, and averaging the statistic over the respective densities. Taking draws from firm i's own cost distribution is trivial since there exists a candidate distribution. The distribution of the rivals' lowest bid can be simulated by taking R random draws for each firm j from $F_a(\mathbf{c})$. For each cost draw, a corresponding bid can be found conditional on the vector with strategy starting values. For each firm i and each draw r, the lowest bid submitted by the rivals is collected to get cost draws from $G_{ia}(y_{ia})$.

Once candidate vectors of CSE strategies are found, the inverse strategy profile can be applied to the observed bids to derive the associated cost for each firm i, $c_{ia}^{(t)} = s_{ia}^{-1}(b_{ia}; \mathbf{d}_{ia}^{(t)})$. The derived cost can thereafter be inserted in equation (12) and an updated vector of structural parameters can be estimated, $(\beta^{(t+1)}, \sigma^{(t+1)})$. Cost draws from the updated joint cost distribution are taken and a solution to the maximization problem is simulated conditional on $\mathbf{d}_a^t$. The inverse of the updated strategy profile $\mathbf{d}_a^{t+1}$ is once more applied to the observed bids to derive updated cost draws, $c_{ia}^{(t+1)}$.

This iterative process is repeated until there is convergence in the estimation procedure. Assuming that the sequence of structural parameters converges as $t \to \infty$, the estimators that represent the solution to the fixed-point problem

[10] The assumption of constant variance across firms and auctions is a simplifying assumption. Since the estimated cost draws are related to the relative bids, it follows from this assumption that the variance in costs is higher for a high-value auction than for an auction of low value. This may e.g. be the case if there is more insecurity, with respect to costs, involved with larger projects.

can be written as

$$\begin{aligned}\hat{\beta} &= \beta^{(t+1)} = \left(\sum \mathbf{x}_{ia}^{'} \mathbf{x}_{ia}\right)^{-1} \sum \mathbf{x}_{ia}^{'} c_{ia}^{(t)} \\ \hat{\sigma}^2 &= \sigma^{2(t+1)} = \frac{1}{A}\sum_{a=1}^{A}\frac{1}{n}\sum_{i=1}^{n}\left(c_{ia}^{(t)} - \mathbf{x}_{ia}^{'}\beta^{(t+1)}\right)^2\end{aligned} \quad , t \to \infty. \tag{13}$$

Since it is not possible to let $t \to \infty$, the relative differences in the parameters between iterations are calculated and evaluated. When the relative difference between round t and round $t+1$ is less than 0.001, the iterative process in finding the structural parameters is being stopped.

The draws from the cost distributions are found by using a so-called Halton sequence. A Halton sequence is defined in terms of a prime number over the unit interval. Compared to random draws (or pseudo-random draws, since nothing produced by a computer is random) the draws produced with the Halton sequence on average have better coverage.[11] Consider the prime 3. The first Halton sequence, s_1, for prime 3 is found by dividing the unit interval into three parts with breaks in 1/3 and 2/3, so that the sequence becomes $\{0, \frac{1}{3}, \frac{2}{3}\}$. The number zero is not a part of a Halton sequence and is later dropped. However, using it initially facilitates the creation of the sequences. In the next iteration, each of the three element from the first sequence is extended according to the following structure: $s_{t+1} = \{s_t + \frac{1}{3^{t+1}}, s_t + \frac{1}{3^{t+1}}, s_t + \frac{1}{3^{t+1}}, s_t + \frac{2}{3^{t+1}}, s_t + \frac{2}{3^{t+1}}, s_t + \frac{2}{3^{t+1}}\}$. The second sequence, s_2, then becomes: $\{0 + \frac{1}{3^{t+1}}, \frac{1}{3} + \frac{1}{3^{t+1}}, \frac{2}{3} + \frac{1}{3^{t+1}}, 0 + \frac{2}{3^{t+1}}, \frac{1}{3} + \frac{2}{3^{t+1}}, \frac{2}{3} + \frac{2}{3^{t+1}}\} = \{\frac{1}{9}, \frac{4}{9}, \frac{7}{9}, \frac{2}{9}, \frac{5}{9}, \frac{8}{9}\}$. In the third iteration, $\frac{1}{3^3}$ and $\frac{2}{3^3}$ are added to each of the nine elements from the first and second sequence and the result is appended to get the third sequence, and so on until the desired number of draws has been reached.

In the simulation, separate draws are taken from each firm's cost distribution. In terms of Halton sequences, this means that a separate prime is used to create separate sequences for each firm in each auction. To avoid correlation between the sequences, the first 100 numbers in each sequence are dropped.

[11] Further details on the Halton procedure, and its advantages, can be found in Train (2003).

As noted above, the Halton draws are defined over a uniform density. To obtain draws from the truncated normal distribution, the inverse of the cumulative distribution is evaluated for each element in the sequence. The shape of the truncated normal density then implies that draws will more frequently be taken from around the mean of the distribution. Consequently, the number of draws needs to be high to get a sufficient amount of draws from the tails of the distribution. A high number of draws is time consuming. Importance sampling can be used to speed up the computational process. By using importance sampling, the total number of draws needed to find convergence can be reduced since some draws are "relocated" to parts of the distribution where they are better needed.[12] The number of Halton draws used in the simulation amounts to $R = 20000$.

The choice of D and starting values for vector $\mathbf{d}$ in the iterative process are important in the estimation. If appropriate starting values are chosen, there is usually fast convergence in the iterative process. In the application, D is restricted to include only those parameter values, so that the constrained strategies are monotonically increasing in cost and they imply non-negative profits, i.e., $s_i(c_i; \mathbf{d}_i) \geq c_i, \ \forall c_i \in [\underline{c}, \overline{c}]$.

Armantier et al. (2005) find that for a number of kinks that divide the strategies into linear segments, K, as low as three, there is very little difference between the CSE and its unconstrained best response with respect to expected utility. In fact, they conclude that with K as low as two, there is robustness to the CSE as an equilibrium concept. Therefore, in this application, the number of kinks is being set to three.[13] The third kink, τ_3, is set equal to the upper

[12] Suppose that we want to simulate the integral $\int m(x)f(x)dx$ by taking R random draws from the truncated normal distribution $f(x)$ and calculating the average over the statistic, i.e., $\frac{1}{R}\sum\limits_{r=1}^{R} m(x_f^r)$. Following the above discussion, most draws will be taken from around the mean of $f(x)$. To relocate draws to the tails of $f(x)$, draws can instead be taken from a uniform distribution $g(x)$ and the integrand be multiplied with the correction factor $g(x)/g(x)$ without changing its value. The following integral is then simulated: $\int \frac{m(x)f(x)}{g(x)} g(x)dx \approx \frac{1}{R}\sum\limits_{i=r}^{R} \frac{m(x_g^r)f(x_g^r)}{g(x_g^r)}$.

[13] Eklöf (2005) also uses three kinks.

support of the cost distributions, $\bar{c}$, and is thus identical for all strategies, across firms and auctions. The positioning of the first two kinks, on the other hand, is determined in the iterative process and varies between auctions, but are the same for the strategies that belong to the same auction. For all firms in auction a, the first and second kink are defined in the following way:

$$\begin{array}{ll} \tau_{a1}^{(t+1)} & = \min\{\mathbf{x}_{ia}^{'}\beta^{(t)}\} - \sigma^{(t)} * p(0.25) \\ \tau_{a2}^{(t+1)} & = \min\{\mathbf{x}_{ia}^{'}\beta^{(t)}\} + \sigma^{(t)} * p(0.75) \end{array} , \forall i = 1, ..., n \tag{14}$$

where $\mathbf{x}_{ia}^{'}\beta^{(t)}$ defines the expected cost for firm i at auction a in iteration t, following the discussion above, and $p(\cdot)$ defines the percentile. Several different rules in the positioning of the kinks have been tested. The used method, however, seems to work well in the application.

5.2 Reduced–Form Specification

To be able to compare the results from the structural estimation with reduced-form results, reduced-form bid estimation results will be presented. When estimating reduced-form models, there is no restriction that all bidders need to be observed. In addition, there is no need to drop observations to facilitate convergence, as discussed above. However, to get results that are comparable with the results from the structural models, the same data, and the same specification, will be used when estimating reduced-form models. This also means that not all information that potentially affects the bid-level decision will be included, and that the models may be misspecified.

The reduced form model is specified in the following way

$$RELBID_{ia} = \mathbf{x}_{ia}'\beta^x + \mathbf{d}_{ia}'\beta^d + \varepsilon_{ia}^b, \tag{15}$$

where $RELBID_{ia}$ represents the dependent variable, the observed bid scaled with the auction mean bid. Subscripts i and a represent firm and auction, respectively. Vector $\mathbf{d}_{ia}$ includes dummy variables and vector $\mathbf{x}_{ia}$ firm-specific variables. The private information that affects firm i's bid at auction a is represented by the residual, ε_{ia}^{b}. The residual has zero expectation and constant variance denoted σ^2.

Not all potential bidders actually submit bids at the auctions. Out of 10 invited firms, slightly above 5 choose to participate in the bidding. If non-participation is not a random decision but rather happens due to some systematic factor such as a high capacity-utilisation level (so that the firms that choose not to bid are all capacity constrained), sample selection bias may be a problem. To test the robustness of the estimated parameters, Heckman's two-step method can be used (1976, 1979). However, no convincing evidence of sample-selection bias was found in Jakobsson (2006) where the same data material is used. Sample-selection bias is therefore assumed not to be a problem in this analysis.

6 Results

In this section, the results from the estimation based on the fixed-point algorithm are presented and discussed, as well as the results from testing the hypothesis of conditional independence.

6.1 Results from the Structural Estimation

The results from the structural estimation is presented in Table A.1. The corresponding reduced-form estimation is also presented to make it possible to investigate the effects of the independent variables on both costs and bids. Results from the fixed-point estimation are presented in the first column of Table A.1, and the results from the reduced-form bid estimation in column (2).

The dependent variable in the cost estimation is the firms' cost draws that are found in the iterative process discussed above. Since the bids used in the structural estimation are relative bids, the cost draws found are related to the relative bids and not to the actual bids.

There is no constant term in the specification, instead fixed effects are estimated for all firms, all years and all regions. The main reason for this is that firms with bidding strategies that cannot be inverted are temporarily dropped in the estimation procedure.[14] Since it is not clear beforehand which these firms are, the selection of a reference firm becomes complicated.

The results presented in the table are as expected. The results show that the firms' cost draws depend positively on the transportation distance, and negatively on plant ownership. The results further indicate that there exists firm-specific information that is not controlled for by $DIST$ and $RPLANT$, since the firm-specific fixed effects are significant. Region and year dummies are also significant, indicating that there exists region- and year-specific information in the costs.

A comparison across columns in Table A.1 shows that costs are more sensitive to variations in the independent variables than the corresponding bids. However, this is by construction of the strategies, but it is interesting to see the magnitude of the differences. As the table shows, the differences are not very big.

Given that the structural model is correctly specified, the residuals can be used to test the hypothesis of conditional independence. Here we assume this to be the case and move on to test if costs are correlated.

However, first the quality of the constrained strategies will be investigated. Since the CSE approach is developed as a tool to approximate the often intractable unconstrained equilibria, a relevant exercise is to investigate the quality of the approximation. Armantier et al. (2005) propose several criteria that are relevant from a game-theoretical perspective in assessing the quality. In this paper, the quality of the CSE is assessed by focusing on one of these criteria, namely the relative difference between the expected utility for a bidder from using the CSE strategy compared to using a unilateral best-response strategy. Consider once more the highest expected utility a bidder can obtain by using his optimal constrained strategy, given that rival firms also use their constrained

[14] From this also follows that the X'X matrix is singular. However, this problem is handled in the estimation by using generalized inversion techniques.

strategies:

$$\widetilde{U}_i(\mathbf{s}^{(K)*}) \approx \frac{1}{R}\sum_{r=1}^{R}(s_i^{(K)*}(c_i^r,\mathbf{d}_i)-c_i^r)K_h\left(y_i^r - s_i^{(K)*}(c_i^r,\mathbf{d}_i)\right). \quad (16)$$

If instead deviating unilaterally and bidding his best response, the expected utility of bidder i can be written as

$$\widetilde{U}_i^{BR}\left(\mathbf{s}_{-i}^{(K)*}\right) \approx \frac{1}{T}\sum_{t=1}^{T}\max_{b_i\in B_i}\frac{1}{R}\sum_{r=1}^{R}(b_i-c_i^t)K_h(y_i^r-b_i)\ , \quad (17)$$

where b_i is the bid that unilaterally maximizes expected utility given the cost c_i and T is the number of draws from firm i's cost distribution that is used in the simulation. In this calculation, $T = 200$.[15]

The relative difference between the expected best-response utility and the CSE expected utility is calculated as

$$Diff_i = \frac{\widetilde{U}_i^{BR}(\mathbf{s}_{-i}^{(K)*}) - \widetilde{U}_i(\mathbf{s}^{(K)*})}{\widetilde{U}_i(\mathbf{s}^{(K)*})}. \quad (18)$$

A value of $Diff_i$ close to zero indicates that there is little difference between the constrained strategy and its unconstrained best response, in terms of expected utility. A low value of $Diff_i$ thus suggests that the incentives to deviate from the CSE are weak. This, in turn, is taken as an indication that the constrained strategies are close to the unconstrained BNE strategies, in terms of expected utility.

When calculating $Diff_i$ for all firms i and all auctions a over the support $[0, 2]$, the results show that, on average, the firms could increase their expected utility by 28.5 percent by bidding their best response instead of using their CSE strategy. The calculated statistic ranges between 7 and 103 percent, the median being 27 percent.

[15] Some of these T draws are not actually draws but interpolations. Interpolation is a fast and easy way of increasing accuracy by making it possible to find best-response bids for costs that are midway between two existing cost draws. The best-response bid for a cost $c_{int} = c_0 + (c_1 - c_0)/2$, is interpolated as $b_{int} = b_0 + (c_{int} - c_0)(b_1 - b_0)/(c_1 - c_0)$.

This result indicates that the strategies are not very good approximations of the unconstrained BNE strategies. Note, however, that the BR strategies calculated in this section give the firms higher expected utilities than what would be the case in a BNE. In the BNE, all firms use their best responses simultaneously, in contrast to the present setup where the BR is calculated under the assumption that the competitors use their CSE strategies.[16]

6.2 Tests for conditional independence

In this sub-section, the results from testing the hypothesis of conditional independence are presented. The hypothesis of conditional independence is tested by estimating Spearman rank correlation coefficients for firms in pairs. The Spearman test is carried out using the residuals from the structural estimation presented in Table A.1 in the appendix. The residuals from the structural estimation are the part of the firms' cost draws that cannot be explained by the variables included in the estimation. Put differently, the residual is the difference between a firm's expected cost and the cost that can be derived from the observed bid, conditional on the equilibrium-strategy function. There exists one vector with residuals for each firm included in the estimation. The residuals in each vector are ranked by giving the residual with the lowest value rank 1 and the second lowest residual rank 2, etc. For the two firms that are tested, the difference in rank for the ith residual is calculated and collected in d_i. The correlation coefficient is calculated as $r_s = 1 - 6\left[\frac{\sum d_i^2}{n(n^2-1)}\right]$, where n is the number of observations in the vector, i.e., the number of auctions in which the firms have participated simultaneously. The correlation coefficients can take a value between -1 and 1.

The hypothesis of conditional independence is tested under the null hypothesis that firms behave independently, i.e., that they take cost draws independently of their competitors. If the tests show a significant correlation, the hypothesis is rejected. The analysis is carried out for all firm pairs with 20 or more si-

[16] Several different specifications, as well as different rules on how to position the kinks, have been explored, without changing the results in any significant way.

multaneous bids. In this case, this means that tests are carried out for 42 firm pairs. The tests for conditional independence for these pairs are presented in Table 2. The results show that the hypothesis of conditional independence can be rejected at the five-percent level of significance in 21 of the 42 tests carried out.

Table 2. Spearman rank correlation coefficients and simultaneous bids

Firms	n	r_s	p-value	Firms	n	r_s	p-value
$(14, 26)$	72	$-.213$	.073	$(26, 55)$	405	$-.275^{***}$	.000
$(14, 43)$	67	$-.003$	.820	$(30, 43)$	36	$.014$	.936
$(14, 44)$	73	$-.231^{**}$	.049	$(30, 44)$	58	$-.638^{***}$	.000
$(14, 50)$	55	$.031$	.822	$(30, 50)$	28	$-.015$	.938
$(14, 55)$	68	$-.217$	.078	$(30, 55)$	53	$-.252$	.069
$(19, 26)$	35	$.208$	.230	$(32, 44)$	20	$-.538^{**}$	.014
$(19, 44)$	32	$.660^{***}$	.000	$(33, 37)$	45	$-.071$	.645
$(19, 50)$	24	$-.535^{***}$	.007	$(33, 44)$	144	$-.340^{***}$	.000
$(19, 55)$	31	$-.611^{***}$	.000	$(33, 55)$	140	$.010$	.907
$(21, 26)$	49	$-.343^{**}$	.012	$(37, 43)$	56	$.002$	.870
$(21, 33)$	40	$-.252$	.117	$(37, 44)$	131	$-.260^{***}$	.003
$(21, 44)$	47	$.311^{**}$	.034	$(37, 50)$	37	$-.389^{**}$	.018
$(21, 55)$	48	$-.198$	.178	$(37, 55)$	98	$-.066$	.521
$(26, 30)$	58	$-.496^{***}$	.000	$(43, 44)$	205	$-.047$	.506
$(26, 32)$	20	$-.759^{***}$	.000	$(43, 50)$	160	$.133$	.094
$(26, 33)$	147	$-.240^{***}$	.004	$(43, 55)$	190	$-.482^{***}$	.000
$(26, 37)$	133	$-.165$	.058	$(44, 49)$	39	$-.542^{***}$	.000
$(26, 43)$	207	$-.091$	.191	$(44, 50)$	194	$-.235^{***}$	.001
$(26, 44)$	451	$.260^{***}$	.000	$(44, 55)$	399	$-.274^{***}$	.000
$(26, 49)$	40	$-.283$	.077	$(49, 55)$	40	$-.200$	.216
$(26, 50)$	196	$-.310^{***}$	.000	$(50, 55)$	184	$-.038$	.612

a. *** and ** represent significance at the 1- and 5-percent levels, respectively.

One of the firm pairs $(19, 50)$ has in the auctions investigated only submitted bids simultaneously a few times per year. Keeping a cartel stable is difficult when bidding is that infrequent. Removing this firm-pair from the list reduces the number of pairs with significant correlation to 20. The tests show that the correlation is negative in 18 out of the 20 tests that turn out significant. Correlation is positive in only two cases. For a total of thirteen firms the hypothesis of conditional independence can be rejected, at least once, in the tests presented above. These are firms 14, 19, 21, 26, 30, 32, 33, 37, 43, 44, 49, 50 and 55. Negative correlation is found for all of these firms.

7 Discussion

The results found when testing the hypothesis of conditional independence imply that about half the firm-pairs tested have cost draws that are not independent, a finding that indicates a bidding behaviour that is not consistent with the model of competitive bidding. Out of the thirteen firms for which it is possible to reject the hypothesis of conditional independence, ten represent the largest firms in the industry, with respect to the value of contracts won.[17] All firms that are alleged with collusive behaviour by the CA, and that are represented in the data, are found to have correlated costs, at least together with one other firm. The three largest firms in the industry, which individually participate in over 84 percent of the auctions included in the sample, are represented with significant correlation between four and ten times in the tests presented above. This finding is in line with the CA'a cartel investigation suggesting that the group of large firms has had a leading role in the collusive behaviour.

When testing the hypothesis of conditional independence, Bajari and Ye (2003) find that most bidding behaviour in the market investigated is consistent with the model of competitive bidding. This conclusion cannot be drawn in this paper. However, the finding that such a large group of firms as thirteen has correlated costs need not be alarming, given that collusion is as widespread as the CA suggests.

Comparing the results found in this paper to the results found in Jakobsson (2006), where the same analysis is carried out based on the same data and reduced-form models, further shows that conditional independence can be rejected for the same group of firms.[18] This indicates that strategic considerations do not appear to be very important in the auctions analysed. This, in turn, suggests that when analysing this particular market, reduced-form models seem to work equally well as the more sophisticated structural models.

[17] See Jakobsson (2006) for an exposition of firms' market shares in terms of contracts won.

[18] In Jakobsson (2006) the hypothesis of conditional independence cannot be rejected for firm 14 when using the whole sample. However, when testing each year separately, conditional independence can be rejected for one year.

However, there are several potiential limitations to the approach used in this paper. Omitted variables may lead us to incorrectly reject the hypothesis of conditional independence. As an example, consider a firm that do not have access to its own production plant. As a consequence, it has to buy asphalt from competitors at the prices offered. Due to high input prices the submitted bid is higher than expected. At the same time the selling firm may bid lower than expected to win the auction. If this behaviour is observed by the firms in the market, but not controlled for in the estimation, negative correlation may be found. However, most firms represented in the above tests have their own production plants, which indicates dependence among firms not to be very important, at least not in this aspect.

Moreover, consider two firms with different capacity-utilisation levels. The firm with capacity constraints may bid high while the firm with a lot of spare capacity may bid low. If capacity usage is observed by the firms, but not controlled for in the estimation, it may lead to incorrectly rejecting the hypothesis of conditional independence.

However, firm-specific dummies are included in the estimation. Therefore, we should be most worried about information on costs that is not correlated with firm-specific effects.

If the structural model is not correctly specified, we may fail to control for strategic considerations. As a consequence, we may falsely conclude that costs that are correlated are the result of collusive behaviour. When deriving firm i's cost draw for auction a in the structural model, it is important that the competitors' cost structures are correctly captured. If there exists information on costs that the firms can observe, but that is not included in the cost estimation, the estimated parameters will be biased. As a consequence, the firms meet competitors with different characteristics in the model than what is actually the case. It follows from this that the bidding strategies found in the structural estimation and, in turn, the cost draws associated with the observed bids, may be incorrect.

As a consequence, firms may have different strategic considerations than those controlled for in the estimation. Omitted variables may therefore lead us to falsely assume that the cost estimates that was derived are free from strategic considerations. In the analysis, normalised bids are used, as well as firm-, year- and region-fixed effects. Therefore, we should be most worried about unobserved information on costs that are not correlated with these variables. This may e.g. happen if three firms meet regularly, and two of them, firms 1 and 2, always buy inputs from the same supplier, while the third does not. If this behaviour is observed in the market, and not controlled for, it may show up as positive correlation between firms 1 and 2.

In addition, being able to reject the hypothesis of conditional independence may also be the result of tacit collusion. The test cannot determine if the observed behaviour is the result of illegal agreements. Therefore, summing up the above discussion, careful judgement should be used in analysing the results.

Another problem from which this analysis may suffer is the theoretical possibility that the hypothesis of conditional independence cannot be rejected, even though collusion exists in the market. Firms may potentially design a bid pattern so that it will not be picked up in the tests of conditional independence. However, in earlier studies where there has been previous evidence of collusive behaviour, it has typically been possible to reject the hypothesis of conditional independence (see e.g. Porter and Zona (1993, 1999) and Bajari and Ye (2003)). A potentially more serious problem with the method used in this paper is that cartels may consist of more than two participants. It may also be the case that bid rigging is combined with bid suppression. This seems e.g. to be the case with the cartel that most likely exists in the Swedish asphalt-paving market. As a result, the method for detecting collusion used in this paper may be unsuccessful in revealing the whole bid behaviour.

8 Conclusions

In this paper, the aim is to assess the issue of collusion in the asphalt-paving industry. More specifically, a necessary and sufficient condition of a competitive

Bayesian-Nash Equilibrium (conditional independence) discussed e.g. in Bajari and Ye (2003) is tested using firms' private costs. For firms to behave competitively, their private cost draws need to be independent, conditional on observable information on costs. If, on the other hand, firms costs are correlated, collusion is one possible explanation.

Firms' costs for carrying out a contract are normally not observed by the researcher. They can, however, be found using structural-estimation techniques. In the industry investigated in this paper, firms are ex ante asymmetric with respect to costs. Solving for equilibrium strategies in games with asymmetric bidders is not trivial, since strategies can normally not be derived analytically.

In the present analysis, an equilibrium concept with bidders that are constrained in the strategy space to a predetermined set of simplified strategies is adopted. When parametrizing these simplified constrained strategies, finding the equilibrium and the structural parameters of bidders' private costs reduces to finding a finite set of parameters, instead of an infinite set of functions as is the case with the Bayesian-Nash Equilibrium.

In earlier research, the hypothesis of conditional independence has been tested using bids. In this research, in contrast, the tests of conditional independence are based on firms' costs, derived using structural estimations. Using costs instead of bids makes it possible to control for firms' strategic considerations when bidding. Not being able to control for strategic considerations may give false indications of collusion in the tests of conditional independence.

The empirical analysis is based on bid data from procurement auctions that took place in Sweden during the 1990's. The main findings are that in about half of the tests that are carried out, the hypothesis of conditional independence can be rejected. Given that the model is correctly specified, the results suggest that there is a large group of firms involved in collusive behaviour. By and large, the same firms that are found in Jakobsson (2006), where reduced-form estimations are used, are found to have dependent costs. If the results are reliable, this further indicates that strategic considerations are not of any great importance in this market.

If this is a more general result, it indicates that reduced-form models can be used in place of structural models when testing the hypothesis of conditional independence, without loss of accuracy. This is especially appealing to e.g. competition authorities, since it might be difficult to find the equilibrium bid functions associated with the structural models. Reduced-form models are less time consuming and easier to handle.

However, the test used in this paper, as any test of collusive behaviour, suffers from limitations. The results found hinge on the assumptions of the auction model, and that the estimated model is correctly specified. If the firms observe information on each other's costs that is not included in the estimation, the hypothesis of conditional independence may erroneously be rejected. Given that the constrained bid strategies estimated in this paper may not be very good in approximating the unconstrained strategies, no strong conclusions can be drawn from the results found.

Moreover, the test cannot determine if the observed pattern is the result of illegal agreements, or if it is caused by tacit collusion. The possibility also exists that, if informed of the structure of the test, a clever cartel can adapt its behaviour so that it is not observed. However, despite the fact that no test for collusion is foolproof, the analysis carried out in this paper could arguably be used as a first step in investigating the existence of collusive behaviour.

A natural extension to the analysis carried out in this paper would be to investigate the social costs induced from the suggested collusive behaviour. This involves estimating a structural model in line with Bajari (2001), where the collusive firms must be selected and the exact collusive behaviour determined. The question of potential damage done by the collusive firms is left open to future research, however.

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9 Appendix

Table A.1. Estimation results, normally distributed costs

Variable	cost	bid
RPLANT	-0.026**	-0.023***
	(0.011)	(0.007)
RELDIST	0.013**	0.011***
	(0.005)	(0.003)
F8	0.164***	0.154***
	(0.029)	(0.018)
F14	0.122***	0.128***
	(0.014)	(0.009)
F15	0.132***	0.129***
	(0.027)	(0.017)
F17	0.128***	0.134***
	(0.035)	(0.022)
F18	-0.057*	0.033*
	(0.030)	(0.019)
F19	0.132***	0.135***
	(0.020)	(0.013)
F21	0.159***	0.150***
	(0.017)	(0.011)
F22	0.188***	0.182***
	(0.031)	(0.020)
F25	0.117***	0.123***
	(0.028)	(0.018)
F26	0.057***	0.102***
	(0.010)	(0.006)
F30	0.154***	0.158***
	(0.016)	(0.010)
F31	0.034	0.079***
	(0.039)	(0.024)
F32	0.053***	0.114***
	(0.026)	(0.016)
F33	0.147***	0.143***
	(0.012)	(0.008)
F34	0.177***	0.164***
	(0.032)	(0.020)
F35	0.139***	0.138***
	(0.027)	(0.017)
F37	0.083***	0.098***
	(0.012)	(0.008)
F43	0.117***	0.121***

Table A.1. Cont.

Variable	cost	bid
	(0.010)	(0.006)
F44	0.091***	0.111***
	(0.010)	(0.006)
F45	0.147***	0.151***
	(0.029)	(0.019)
F49	0.115***	0.142***
	(0.018)	(0.012)
F50	0.107***	0.115***
	(0.010)	(0.006)
F54	0.112***	0.116***
	(0.035)	(0.022)
F55	0.110***	0.122***
	(0.007)	(0.005)
$\bar{R}^2$	0.06	0.07
F - statistic	3.72	4.39
DF	2398	2398
N	2440	2440

a. Dependent variable in cost equation is the estimated cost draws.
b. Dependent variable in bid equation is RELBID.
c. Region- and year-fixed effects are included in the estimations
d. Standard errors in parenthesis.
e. ***, **, and * denote significance at the 1, 5 and 10-percent level.