On the Role of Patience in Collusive Bertrand Duopolies*

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Abstract. This paper analyzes the role of patience in a repeated Bertrand duopoly where firms bargain over which collusive price and market share to implement. It is shown that the least patient firm's market share is not monotone in its own discount factor.

JEL: C72, D43, L11, L41

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1 Introduction

Traditionally, most theoretical investigations of repeated interaction and collusion have focused on determining the set of outcomes that can be sustained as subgame perfect equilibria.[1] This has led to an embarrassment of riches -almost everything is an equilibrium. However, little attention has been given to the obvious but intricate questions regarding *how* agreement is reached and *which* equilibria firms select.

An exception is Harrington (1989) who investigates collusion in an infinitely repeated Bertrand game where firms have different discount factors

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[1]See Feuerstein (2005) for a recent survey.

and choose both price and market share in every period.[2][3] He uses the Nash bargaining solution to select a collusive equilibrium and finds that a firm's market share is monotonically decreasing in its own discount factor and moreover if firms' discount factors are above 0.5 the equilibrium is symmetric and unaffected by marginal changes in discount factors. However, using an axiomatic approach to model the bargaining between firms has its weaknesses. It is hard to assess the validity of the axioms and moreover, within this approach one cannot address concerns regarding the bargaining protocol, (Osbourne and Rubinstein 1990, p. 69). One way to address these issues is, as first suggested by Nash (1953), to take an alternative approach and model it as a strategic game. Moreover, the two approaches should be seen as complementary (Nash 1953, p. 129).

In this paper we take a similar approach as in Harrington (1989), but we model the bargaining as an alternating-offer game (Binmore 1987, Rubinstein 1982) where firms take turns to propose prices and market shares. In this setting there are two effects from differences in discount factors. On the one hand, the most patient firm is less eager to settle quickly on an agreement and can use this to propose an agreement that favors it. On the other hand, in the infinitely repeated Bertrand game the less patient firm has a potential advantage since it has a more restrictive incentive compatibility constraint and must therefore be allocated a higher stage game profit to be induced to collude. Interestingly these two effects work in opposite direction and ex ante it is hard to determine how this will affect outcomes. We investigate equilibrium agreements when the length of bargaining periods are arbitrarily short and contrary to a statement in Harrington (1989) (p. 269) we find that his results cannot be obtained from an alternating-offer bargaining game. More specifically we find that equilibrium market shares will generally be different from his and moreover, the least patient firm's market share will not be monotone in its own discount factor. However, in line with the previous study we find that the most patient firm will earn the

[2]IO models of repeated interaction have focused on either price or quantity competition. However, in a recent study of about twenty cartel decisions made by the European Commission, Harrington (2006) notes that almost every cartel coordinated on both issues.

[3]Discount factors are important determinants for firms trying to collude and there are many reasons why firms may have different discount factors. For instance, firms might have different probabilities of going bankrupt. See Merton (1974) for a seminal paper on this subject.

highest profit.

2 The Infinitely Repeated Bertrand Game

2.1 The Stage Game

Consider an industry with two firms producing a homogenous product at identical constant marginal costs. Without loss of generality we normalize marginal costs to zero. Market demand is given by a continuous and bounded function $D : \mathbb{R}_+ \to \mathbb{R}_+$ such that there exists $\bar{p} > 0$ with $D(p) = 0$ if and only if $p \geq p^{MAX}$ and $D(p)$ is decreasing on the interval $[0, p^{MAX}]$. Finally assume that there exists a unique industry monopoly price p^m.

Firms compete by simultaneously choosing prices $p_i \in P = \mathbb{R}_+ \ i = 1, 2$. The firm that sets the lowest price serves the entire market. A standard assumption in textbook treatments on Bertrand games is that, in case of a price tie, demand is allocated equally among firms. However, in line with Harrington (2006) we allow for unequal market shares in this case. To model this we let each firm choose a price p_i and a market share $s_i \in [0, 1]$. In case of a price tie and $s_1 + s_2 = 1$ firms get their quoted share. However, if there is a price tie and $s_1 + s_2 \neq 1$ we assume that firms share the market equally.[4] Formally individual demand equals

$$D_i(p_i, p_j, s_i, s_j) = \begin{cases} 0 & \text{if } p_i > p_j \\ D(p_i) & \text{if } p_i < p_j \\ s_i D(p_i) & \text{if } p_i = p_j \text{ and } s_1 + s_2 = 1 \\ \frac{D(p_i)}{2} & \text{if } p_i = p_j \text{ and } s_1 + s_2 \neq 1 \end{cases} \tag{1}$$

We let $\pi_i(p_i, p_j, s_i, s_j) = p_i D_i$ denote the stage game profit of firm i. Given the above assumptions both firms set $p_i = 0$ and hence earn zero profit in any Nash equilibrium.[5]

2.2 The Repeated Game

Now consider the situation where the stage game described above is repeated an infinite number of times, i.e. an infinitely repeated Bertrand

[4] A similar approach is used in Athey and Bagwell (2001).

[5] Contrary to standard Bertrand games there is actually a continuum of equilibria in this game. However, every equilibrium leads to zero profits.

game (IRBG), and after each stage both p_i and s_i are observed by both firms. Let $\sigma = (\sigma_1, \sigma_2)$ denote a strategy profile.[6] The objective for each firm is to maximize $\Pi_i = \sum_{t=1}^{\infty} \delta_i^{t-1} \pi_i(\sigma(t))$ where $\delta_i \in (0,1)$ is the firm specific discount factor. We may, w.l.o.g. assume that $\delta_1 \geq \delta_2$. Henceforth we will focus on prices in $[0, p^m]$ and market shares such that $s_1 + s_2 = 1$.

We assume that firms are restricted to use a slightly modified version of the grim trigger strategy introduced in Friedman (1971). Firm i uses the following strategy, where $\bar{p}, \bar{s}_1$ and $\bar{s}_2$ are the collusive price and market shares: Set $p_i = \bar{p}, s_i = \bar{s}_i$ if $t = 1$ or if $t > 1$ and $p_j = \bar{p}, s_j = \bar{s}_j, j = 1, 2$ in every previous period, otherwise set $p_i = 0, s_i = \bar{s}_i$.[7]

A necessary and sufficient condition for a pair of such strategies to be a subgame perfect equilibrium (SPE) is

$$\frac{1}{1-\delta_i} \bar{p} \bar{s}_i D(\bar{p}) \geq \bar{p} D(\bar{p}). \tag{2}$$

At $\bar{p} = 0$ the condition will always hold. But if $\bar{p} > 0$, inequality (2) simplifies to

$$\bar{s}_i \geq (1 - \delta_i) \equiv \underline{s}_i. \tag{3}$$

Hence (3) is the incentive compatibility (IC) constraint on each firm's market share. The bound $\underline{s}_i$ is decreasing in δ_i which means that a more patient firm requires a smaller market share to fulfill its IC constraint. The set of SPE allocations $N(\delta)$ is thus

$$N(\delta) \equiv \{(p, s_1, s_2) \in [0, p^m] \times \Delta | p \in (0, p^m] \text{ and } s_i \geq (1 - \delta_i), \text{ or } p = 0\},$$

where $\delta = (\delta_1, \delta_2)$ and Δ is the one dimensional unit simplex. Because setting $\bar{p} = 0$ in every subgame is an SPE independent of discount factors, $N(\delta) \neq \emptyset$. However, for $N(\delta)$ to include other elements we must have $\delta_1 + \delta_2 \geq 1$.[8]

Let $V(\delta)$ denote the set of payoffs that are sustainable as SPE payoffs.

[6] As usual σ_i is an infinite sequence of maps from the set of histories to the set of stage game actions.

[7] Setting $s_i = \bar{s}_i$ in the punishment phase is without loss of generality since in the punishment phase both firms earn zero profit.

[8] By summing up inequality (3) for $i = 1, 2$ and using $\bar{s}_1 + \bar{s}_2 = 1$ we get the stated result.

3 Equilibrium Analysis

$V(\delta)$ defines an SPE "slice" of the "cake" whose elements can be attained by choosing an appropriate price and market share allocation from $N(\delta)$. In this paper we assume that firms meet before the IRBG begins to negotiate over which equilibrium to implement. We model the equilibrium selection process as a generalized alternating-offer game (Binmore 1987).

Whereas prices might only be updated once a month, perhaps due to menu costs, it could be argued that the time elapsed between an offer and a counter offer in a bargaining session are considerably shorter. Accordingly we let the length of time periods in the bargaining game approach zero. Once a proposal is accepted the bargaining game ends and the IRBG begins, i.e. we do not allow renegotiation. We assume that firms only make proposals that can be implemented as an SPE in the IRBG. Formally let $d(\delta) = (\bar{p}, \bar{s}_1, \bar{s}_2, \bar{k})$ be a limit SPE outcome of the bargaining game where $\bar{p}$ is the equilibrium price, $\bar{s}_i$ equilibrium market shares and $\bar{k}$ the time period when agreement was reached. We now state the equilibrium agreements, where $\lambda^0 = \frac{1}{\ln \delta_1 + \ln \delta_2}$.

Proposition 1 *For $\delta_1 \geq \delta_2$ and $\delta_1 + \delta_2 \geq 1$ the limit SPE outcomes $d(\delta)$ are as follows:*

(a) If $\underline{s}_2 \leq \lambda^0 \ln \delta_1$ then $d(\delta) = (p^m, \lambda^0 \ln \delta_2, \lambda^0 \ln \delta_1, 1)$.

(b) If $\underline{s}_2 > \lambda^0 \ln \delta_1$ then $d(\delta) = (p^m, 1 - \underline{s}_2, \underline{s}_2, 1)$.

Proof.

We start by showing that $\underline{s}_1 \leq \lambda^0 \ln \delta_2$ always holds, implying that cases *(a)* and *(b)* are exhaustive.

Lemma 1 *If $\delta_1 \geq \delta_2$ and $\delta_1 + \delta_2 \geq 1$ then $\underline{s}_1 \leq \lambda^0 \ln \delta_2$.*

Proof.

Assume, to get a contradiction, that $\delta_1 \geq \delta_2$, $\delta_1 + \delta_2 \geq 1$ and $\underline{s}_1 > \lambda^0 \ln \delta_2$ hold. Since $\delta_1 \geq \delta_2$ we must have $\lambda^0 \ln \delta_2 \geq \lambda^0 \ln \delta_1$ and $\underline{s}_2 \geq \underline{s}_1$ which implies $\underline{s}_2 > \lambda^0 \ln \delta_1$. Now, summing up $\underline{s}_i$ and using the fact that $\underline{s}_i = 1 - \delta_i$ we get $(1 - \delta_1) + (1 - \delta_2) > \lambda^0 \ln \delta_2 + \lambda^0 \ln \delta_1$. But this implies $\delta_1 + \delta_2 < 1$, a contradiction. Hence, $\underline{s}_1 \leq \lambda^0 \ln \delta_2$. ∎

Since $V(\delta)$ is a convex set we can use the results in Binmore (1987) stating that, in the limit as the length of bargaining periods approach zero,

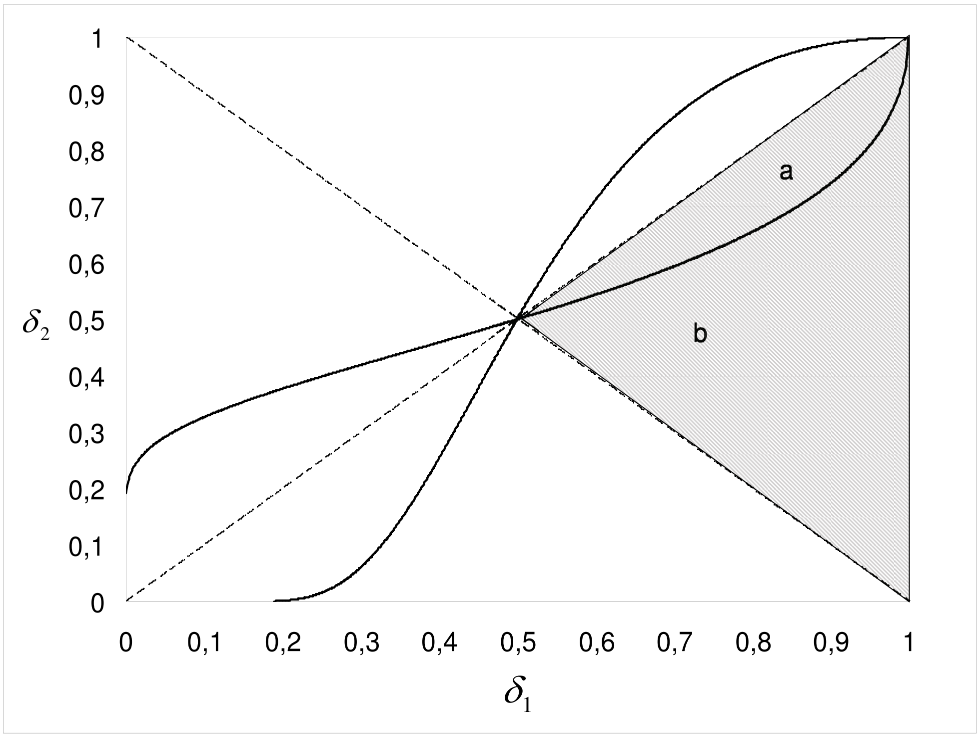

Figure 1: Extension of case *(a)* and *(b)*

the unrestricted market sharing agreement will be given by $\bar{s}_i = \lambda^0 \ln \delta_j$. Moreover, the agreements will be Pareto which in our model implies that $\bar{p} = p^m$ and $\bar{k} = 1$. This establishes case *(a)*. In case *(b)* firm 2's IC constraint will be be binding and in this case it is easy to see that the solution will be given by $d(\delta) = (p^m, 1 - \underline{s}_2, \underline{s}_2, 1)$. ■

There are two interesting features to note: First, equilibrium agreements are efficient. Second, in case *(a)* of the proposition the market shares are determined by firms' relative bargaining power, whereas in case *(b)* it is determined by firm 2's IC constraint. The extension of the two cases in $\delta_1 \times \delta_2$ space is illustrated in Figure 1 and the shaded region indicates where the two assumptions $\delta_1 \geq \delta_2$ and $\delta_1 + \delta_2 \geq 1$ are met.

An interesting corollary to Proposition 1 is that $\bar{s}_2$ will not be monotone in δ_2. To illustrate this point we have in Figure 2 fixed δ_1 to 0.7 and plot $\bar{s}_2$ as a function of δ_2. The non-monotonicity stems from the fact that for sufficiently low δ_2 agreements will be determined by firm 2's IC constraint which is decreasing in δ_2, whereas for higher values it will be determined by

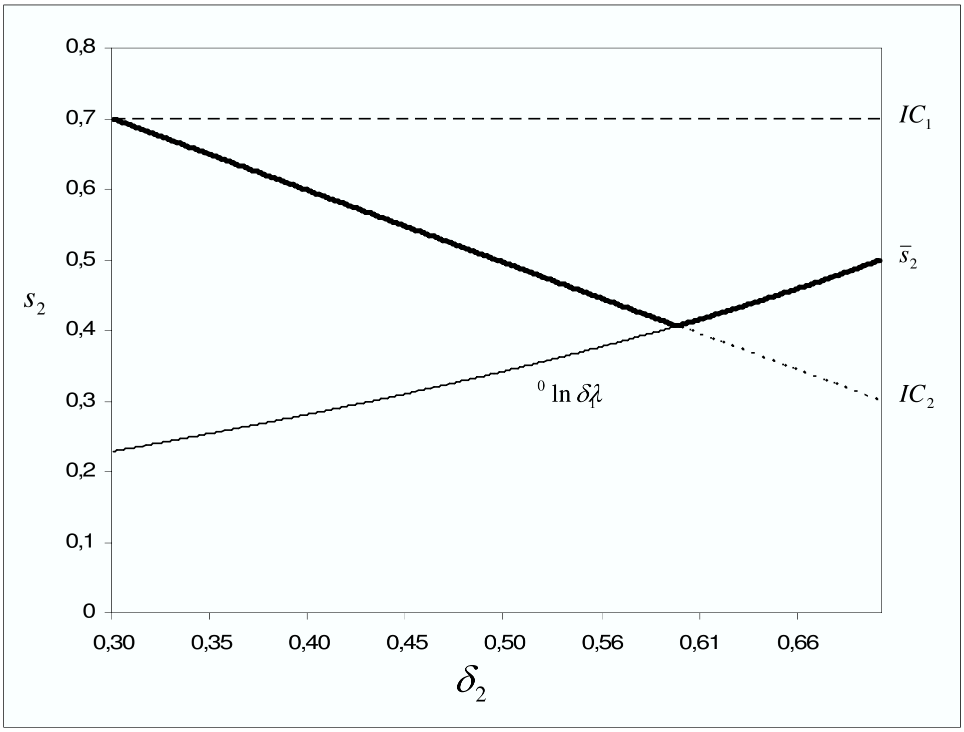

Figure 2: $\bar{s}_2$ when $\delta_1 = 0.7$.

firm 2's relative bargaining power which is increasing in δ_2. By contrast, it is easy to see from Proposition 1 that $\bar{s}_1$ will be increasing in δ_1.

3.1 Equilibrium Market Shares and Profits

Proposition 1 shows a quite intricate relationship between discount factors and equilibrium market shares and it is easy to find examples where firm 2 has a higher equilibrium market share and vice versa. More specifically, in case *(a)* of Proposition 1 it is obvious that $\bar{s}_1 \geq \bar{s}_2$. However, as can be seen in Figure 2 this is not always the case in *(b)*. This naturally raises the question concerning which firm will have the highest profits in equilibrium.

Proposition 2 *If* $\delta_1 \geq \delta_2$ *and* $\delta_1 + \delta_2 \geq 1$ *then* $\Pi_1 \geq \Pi_2$.

Proof. In order to prove the proposition we need to consider the two cases *(a)* and *(b)* in Proposition 1.

In case *(a)* the result is obvious since $\bar{s}_1 \geq \bar{s}_2$ and $1/(1-\delta_1) \geq 1/(1-\delta_2)$. In case *(b)* it suffices to note that firm 2's IC constraint will be binding, hence $\Pi_2 = p^m D(p^m)$ and because firm 1 could also get this by deviating, which he will not since its IC constraint is met, we must have $\Pi_1 \geq \Pi_2$. ■

A corollary to Proposition 2 is that if $\delta_1 > \delta_2$ and $\delta_1 + \delta_2 > 1$ then $\Pi_1 > \Pi_2$. Proposition 2 is interesting, especially in light of the results in the previous section. Even if the less patient firm has a potential advantage the more patient firm always receives a higher equilibrium profit. Moreover, the ordering of profits in Proposition 2 perfectly corresponds to Proposition 4 in Harrington (1989).

In regards to the non-monotonicity results in the previous section it is also interesting to investigate how Π_2 reacts to changes in δ_2. In case *(a)* a marginal increase in δ_2 will lead to a an increase in Π_2 since firm 2 will now get a higher market share and have a higher valuation of future profit streams. However, in case *(b)* a marginal increase in δ_2 will leave Π_2 unaffected because the increase in the valuation of future profit streams will be exactly offset by the decrease in market share. Hence in this situation firm 2 would have no marginal incentive to increase its discount factor even though it would generally make collusion easier.

4 Concluding Remarks

This paper analyzes which prices and market shares are chosen within an infinitely repeated duopoly framework where firms have different discount factors. We model the equilibrium selection as a strategic game and characterize a unique and efficient equilibrium. In equilibrium the most patient firm always earn the highest profit. Moreover we show, contrary to what was previously believed (cf. Harrington 1989 p. 296), that a strategic approach will give different market share predictions than an axiomatic approach. More specifically, the least patient firm's equilibrium market share is not monotone in its own discount factor. The difference is driven by the fact that the Nash bargaining solution in Harrington (1989) is independent of discount factors. [9]

It was suggested that the paper should require that agreements where renegotiation-proof (Farrell and Maskin 1989). However, it is easily proved that all stationary weakly renegotiation proof strategies in the IRBG would entail setting $p = 0$ in every period, hence there would be no benefit from collusion. However, this seems to contradict empirical data (Harrington 2006).

There are many ways to further the analysis performed in this paper. As suggested by Lehrer and Pauzner (1999) there might be gains to intertemporal trade, thus one line of further research is to extend the analysis to allow for non-stationary strategies. It would also be interesting to investigate how asymmetric information about discount factors would affect equilibrium outcomes. Earlier results (Rubinstein 1985) suggests that this could introduce inefficiencies in the bargaining game because a higher discount factor increases the bargaining power and hence firms have an incentive to overstate it. However, in our setting it also lowers the IC constraint and it is thus not clear that previous results continue to hold.

On a more fundamental level, we took the bargaining protocol as given. However, as a response to empirical findings, e.g. Levenstein and Suslow (2002), we think that more research has to focus on studying this phenomenon in order to understand how collusive agreements are reached.

[9]To see this it suffices to note that, since he is restricting attention to stationary strategies, the Nash bargaining solution is given by $\max \prod_{i=1}^{n} \frac{1}{(1-\delta_i)} \pi_i$ for some profit π_i of the stage game. This can be re-written as $\prod_{i=1}^{n} \frac{1}{(1-\delta_i)} \max \prod_{i=1}^{n} \pi_i$, thus the effect of discount factors is absent.

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